ENRICHED CATEGORIES OF COALGEBRAS

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ABSTRACT. Let $\mathcal{V}$ be a monoidal category with underlying ordinary category $\mathcal{V}_0$. For any $\mathcal{V}$-endofunctor $T : \mathcal{A} \to \mathcal{A}$, define a T-coalgebra as a pair (A, τ_A) consisting of an object A in $ob\mathcal{A}$ together with a $\mathcal{V}_0$-morphism $\tau_A : I \to \mathcal{A}(A, TA)$, where I is the unit. A is called the underlying object of the T-coalgebra (A, τ_A) and τ_A its coalgebra structure. Assuming $\mathcal{V}$ is symmetric and admits equalizers, we prove that T-coalgebras form a $\mathcal{V}$-category denoted $\mathcal{A}_T$. As a result, the correspondence $U_T : ob\mathcal{A}_T \to ob\mathcal{A}$ that assigns to each T-coalgebra (A, τ_A) its underlying object A is a $\mathcal{V}$-functor. If more $\mathcal{V}$ is closed then we prove that the underlying $\mathcal{V}$-functor $U_T : \mathcal{A}_T \to \mathcal{A}$ creates weighted colimits and weighted limits that T preserves.

REFERENCES

[1] J. Adamek, *Introduction to coalgebra*, Theory and Applications of Categories **14** (2005), 157-199.

[2] M. Barr, *Terminal coalgebras in well-founded set theory*, Theoretical Computer Science **114** (1993), 299-315.

[3] F. Borceux, *Handbook of Categorical Algebra*, Encyclopedia of Mathematics and its Applications Volume 2, Cambridge University Press (1994).

[4] B.J. Day and G.M. Kelly, *On toplogical quotient maps preserved by pullbacks or products*, Pro. Cambridge Phil. Soc. **67** (1970), 553-558.

[5] P. Etingof, S. Gelaki, D. Nikshych, V. Ostrik, *Tensor Categories*, Mathematcal Surveys and Monographs, Vol. 205, American Mathematical Society, 2015.

[6] G.M. Kelly, *Basic concepts of enriched category theory*, Reprints in Theory and Applications of Categories **10** (2005), 1-143.

[7] S. Mac Lane, *Categories for the working mathematician*, Graduate Texts in Mathematics, vol. 5, Springer-Verlag 1971.

[8] N. Popescu, and L. Popescu, *Theory of categories*, Editura Academiei Bucuresti, Romania, 1979.

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